Topologically Twisted SUSY Gauge Theory, Gauge-Bethe Correspondence and Quantum Cohomology

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Current Topics in String Theory: Conformal Field Theories

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Relations among

Integrable System

Supersymmetric Gauge Theory

Quantum Cohomology

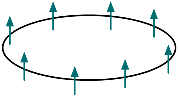
Today:

Integrable System	$\rightsquigarrow$	$\mathrm{XXX}_{1/2}$ Heisenberg Spin Chain Model
Supersymmetric Gauge Theory	$\rightsquigarrow$	2d $\mathcal{N} = (2,2)$ theory with an adjoint chiral multiplet in addition to the same number of fund. and anti-fund. chiral multiplet
Quantum Cohomology	$\rightsquigarrow$	Equivariant Quantum Cohomology of Cotangent Bundle of Grassmanian

Outline

1. Introduction
2. $\mathrm{XXX}_{1/2}$ Heisenberg Spin Chain Model
   - Algebraic Bethe Ansatz
3. 2d $\mathcal{N} = (2,2)$ Theories
   - Properties
   - Gauge-Bethe Correspondence
4. Quantum Cohomology of Cotangent Bundle of Grassmanian
5. Summary

$\mathbf{XXX}_{1/2}$ Heisenberg Spin Chain Model



- Degree of freedom: Spin 1/2 of $SU(2)$ at each M lattice sites
- Hamiltonian: $H = J\sum_{a=1}^{M}\left(S_a^x S_{a+1}^x + S_a^y S_{a+1}^y + S_a^z S_{a+1}^z\right)$
  - $\vec{S}_a = \frac{i}{2}\vec{\sigma}_a$
  - quasi-periodic boundary condition: $\vec{S}_{M+1} = e^{\frac{i}{2}\vartheta\sigma_3}\vec{S}_1 e^{-\frac{i}{2}\vartheta\sigma_3}$
  - $J > 0$: ferromagnet, $J < 0$: anti-ferromagnet
  - position ν_i of the lattice sites with respect to symmetric round lattice configuration
- Hilbert space: $\mathcal{H}_M^{\vartheta} = \underbrace{\mathbb{C}^2 \otimes \ldots \otimes \mathbb{C}^2}_{M}$
- $[S^z, H] = 0$ for any ϑ where $S^z = \sum_{a=1}^{M} S_a^z$
- N-particle sector: subspace $\mathcal{H}_M^{\vartheta,N} \subset \mathcal{H}_M^{\vartheta}$ where $S^z = N - \frac{1}{2}M$
  - pseudo-vacuum: $|0\rangle = |\underbrace{\downarrow\ldots\downarrow}_{M}\rangle$ annihilated by all S_a^-

Algebraic Bethe Ansatz for $\mathbf{XXX}_{1/2}$

- Monodromy Matrix: $\mathrm{T}(\lambda) = \begin{pmatrix} \mathrm{A}(\lambda) & \mathrm{B}(\lambda) \\ \mathrm{C}(\lambda) & \mathrm{D}(\lambda) \end{pmatrix}$
  - acting on the 2-dimensional auxiliary space
  - λ are called spectral parameter

- Transfer Matrix: $\tau(\lambda) = \mathrm{A}(\lambda) + \mathrm{D}(\lambda)$
  - $\tau(\lambda) = \mathrm{A}(\lambda) + e^{i\vartheta}\mathrm{D}(\lambda)$ for quasi-periodic boundary condition

- R-Matrix: $\mathrm{R}(\lambda, \mu) = \begin{pmatrix} f(\mu,\lambda) & 0 & 0 & 0 \\ 0 & g(\mu,\lambda) & 1 & 0 \\ 0 & 1 & g(\mu,\lambda) & 0 \\ 0 & 0 & 0 & f(\mu,\lambda) \end{pmatrix}$
  - $f(\mu,\lambda) = 1 + \frac{ic}{\mu-\lambda}, \quad g(\mu,\lambda) = \frac{ic}{\mu-\lambda}$
  - auxiliary parameter c, which can take arbitrary value
  - acting on the 2×2-dimensional auxiliary space

- Yang-Baxter Equation:

$$R_{12}(\lambda,\mu)R_{13}(\lambda,\nu)R_{23}(\mu,\nu) = R_{23}(\mu,\nu)R_{13}(\lambda,\nu)R_{12}(\lambda,\mu)$$

  acting on auxiliary space $V_1 \otimes V_2 \otimes V_3$ where R-matrix R_{ab} acts on $V_a \otimes V_b$

  $\rightsquigarrow \quad R_{ab}(\lambda,\mu)(T_a(\lambda) \otimes T_b(\mu)) = (T_b(\mu) \otimes T_a(\lambda))R_{ab}(\lambda,\mu)$

  $\Rightarrow$ providing the commutation relations of the matrix elements, A, B, C, and D, of monodromy matrix T

- One can show $[\tau(\lambda)\,,\,H] = 0$
  - $\tau(\lambda)$ is a generating function of conserved charges

- Pseudo-vacuum:
  $A(\lambda)|0\rangle = a(\lambda)|0\rangle, \quad D(\lambda)|0\rangle = d(\lambda)|0\rangle, \quad C(\lambda)|0\rangle = 0$
  where $a(\lambda)$ and $d(\lambda)$ are called vacuum eigenvalues

- Bethe Eigenstates: $|\Psi_N(\lambda)\rangle = \prod_{a=1}^{N} B(\lambda_a)|0\rangle$
  - dual vector: $\langle\Psi_N(\lambda)| = \langle 0|\prod_{a=1}^{N} C(\lambda_a)$

- Bethe Ansatz Equation:

$$\prod_{j=1}^{M} \frac{\lambda_a - \nu_j - i\frac{c}{2}}{\lambda_a - \nu_j + i\frac{c}{2}} = e^{i\vartheta} \prod_{\substack{b=1 \\ b\neq a}}^{N} \frac{\lambda_b - \lambda_a + ic}{\lambda_b - \lambda_a - ic} \,,$$

- Norm of Bethe Eigenstates $\langle \Psi_N(\lambda) | \Psi_N(\lambda) \rangle$: [Korepin '82]

$$\sum_{(\lambda)\in P_{\mathrm{XXX}}} \langle \Psi_N(\lambda) | \Psi_N(\lambda) \rangle^{-1} = \sum_{(\lambda)\in P_{\mathrm{XXX}}} (ic)^{-N} \prod_{a\neq b} \frac{\lambda_a - \lambda_b}{\lambda_a - \lambda_b + ic} \prod_{a=1}^{N} \prod_{j=1}^{M} \frac{1}{(\lambda_a - \nu_j - i\frac{c}{2})(\lambda_a - \nu_j + i\frac{c}{2})} \ \det(\widetilde{\varphi}')^{-1}$$

where

$$\widetilde{\varphi}'_{ab} = \delta_{ab}\left[\left(\sum_{l=1}^{M} \left(\frac{ic}{(\lambda_a - \nu_l - i\frac{c}{2})(\lambda_a - \nu_l + i\frac{c}{2})} \right) + \sum_{s=1}^{N} \left(\frac{2ic}{(\lambda_s - \lambda_a + ic)(\lambda_a - \lambda_s + ic)} \right) \right) - \left(\frac{2ic}{(\lambda_a - \lambda_b + ic)(\lambda_b - \lambda_a + ic)} \right)\right]$$

P_{XXX}: the set of the independent solutions of $(\lambda) = (\lambda_1, \cdots, \lambda_N)$ satisfying the Bethe ansatz equations.

2d $\mathcal{N} = (2,2)$ Theories

- SUSY representations
  - Vector Multiplet: $V = (v_\mu, \sigma, \overline{\sigma}, \lambda, \overline{\lambda}, D)$
  - Chiral Multiplet: $Q = (\phi, \psi, F)$
  - Anti-Chiral Multiplet: $\widetilde{Q} = (\widetilde{\phi}, \widetilde{\psi}, \widetilde{F})$

- Theory we are interested in:
  - gauge group: $U(N)$
  - N_f fund. and N_f anti-fund. chiral multiplet, $Q, \widetilde{Q}$ $\rightsquigarrow$ $SU(N_f)_Q \times SU(N_f)_{\widetilde{Q}}$ flavor symmetries
  - an adjoint chiral multiplet Φ $\rightsquigarrow$ $U(1)_D$ flavor symmetry

	$U(N_c)$	$SU(N_f)_Q$	$SU(N_f)_{\widetilde{Q}}$	$U(1)_D$	$U(1)_R$
Q	N_c	$\overline{N_f}$	$\mathbf{1}$	$-1/2$	r_1
$\widetilde{Q}$	$\overline{N}_c$	$\mathbf{1}$	N_f	$-1/2$	r_2
Φ	adj	$\mathbf{1}$	$\mathbf{1}$	1	R

- Effective twisted superpotential $\widetilde{\mathcal{W}}_{\text{eff}}$:

$$\widetilde{\mathcal{W}}_{\text{eff}} = \tau \sum_{a=1}^{N_c} \sigma_a - \frac{1}{2} \sum_{1 \le a < b \le N_c} (\sigma_a - \sigma_b) - \frac{1}{2\pi i} \Bigg[\sum_{a=1}^{N_c} \sum_{i=1}^{N_f} (\sigma_a - m_i^y - \frac{1}{2} m^z)(\log(\sigma_a - m_i^y - \frac{1}{2} m^z) - 1) + (-\sigma_a + m_i^{\tilde{y}} - \frac{1}{2} m^z)(\log(-\sigma_a + m_i^{\tilde{y}} - \frac{1}{2} m^z) - 1) + \sum_{a,b=1}^{N_c} (\sigma_a - \sigma_b + m^z)(\log(\sigma_a - \sigma_b + m^z) - 1) \Bigg] .$$

- Condition for supersymmetric vacua:

  From the effective superpotential, the condition for supersymmetric vacua, $\exp(2\pi i \partial_{\sigma_a} \widetilde{\mathcal{W}}_{\text{eff}}) = 1$, is given by

$$\prod_{i=1}^{N_f} \frac{(\sigma_a - m_i^y - \frac{1}{2} m^z)}{(\sigma_a - m_i^{\tilde{y}} + \frac{1}{2} m^z)} = (-1)^{N_f} e^{2\pi i \tau} \prod_{b \neq a}^{N_c} \frac{\sigma_b - \sigma_a + m^z}{\sigma_b - \sigma_a - m^z} .$$

  - the mass parameter and flux of Cartan of global symmetries

$$SU(N_f)_Q : (m_i^y, n_i), \quad SU(N_f)_{\widetilde{Q}}, : (m_i^{\tilde{y}}, \tilde{n}_i), \quad U(1)_D : (m^z, l).$$

  - complexified FI parameter τ: $\tau = \frac{\theta}{2\pi} + i\xi$
  - This is one of key ingredient in Gauge-Bethe Correspondence

- 2d $\mathcal{N} = (2,2)$ theory on S^2
  - SUSY theory on curved manifold can be understood by considering off-shell supergravity background, which has non-trivial generalized Killing spinors
  - Among others, we are interested in the case where there is nonzero magnetic flux for background $U(1)_R$ gauge field $A^{(R)}$

$$\frac{1}{2\pi}\int dA^{(R)} = -1$$

  - Above case corresponds to actually topological A-twisted theory.

    [Closset-Cremonesi '14]

- Partition function and Correlation function
  - They are obtained by using localization technique in [Closset-Cremonesi-Park '15, Benini-Zaffaroni '15]
  - Choosing poles in contour integral is prescribed by Jeffery-Kirwan residue formula

$$Z^{2d} = \frac{1}{N_c!} \sum_{\vec{k} \in \mathbb{Z}^{N_c}} q^{\sum_{a=1}^{N_c} k_a} \oint \prod_{a=1}^{N_c} \frac{d\sigma_a}{2\pi i} Z_{\text{total}}^{\text{1-loop}}(k)$$

$$\text{where } Z_{\text{total}}^{\text{1-loop}}(k) = Z_{\text{vector}}^{\text{1-loop}}(k)\, Z_{Q}^{\text{1-loop}}(k)\, Z_{\widetilde{Q}}^{\text{1-loop}}(k)\, Z_{\Phi}^{\text{1-loop}}(k)\,.$$

- Gauge Invariant Operators $\mathcal{O}(\sigma)$
  - σ is scalar component of vector multiplet
  - $\mathcal{O}(\sigma)$ is provided by symmetric polynomials of σ_a, $a = 1, \ldots, N_c$
  - When the Omega deformation is turned off on S^2, gauge invariant operator $\mathcal{O}(\sigma)$ can be put any place on S^2

- Partition function of A-twisted 2d $\mathcal{N}=(2,2)$ theory: [HJC-Yoshida '16]

$$
\begin{aligned}
Z^{2d} =& (-1)^{\frac{N_c(N_c+1)}{2}} (m^z)^{N_c(R-1-l)} \sum_{\sigma\in P_{2d}} \det(\mathcal{M}^{2d})^{-1} \prod_{a\neq b}(\sigma_a-\sigma_b)(\sigma_a-\sigma_b+m^z)^{R-1-l} \\
&\times \prod_{a=1}^{N_c}\prod_{i=1}^{N_f}(\sigma_a - m_i^y - \frac{1}{2}m^z)^{r_1-1+n_i+\frac{1}{2}l}(-\sigma_a+m_i^{\widetilde{y}}-\frac{1}{2}m^z)^{r_2-1-\widetilde{n}_i+\frac{1}{2}l}
\end{aligned}
$$

where

$$
\begin{aligned}
P_{2\mathrm{d}} &:= \{(\sigma_1,\cdots,\sigma_{N_c}) \,|\, \exp(2\pi i\partial_{\sigma_a}\widetilde{\mathcal{W}}_{\mathrm{eff}}) = 1 \text{ for all } a=1,\ldots,N_c\} \\
\mathcal{M}^{2d}_{ab} &:= (-2\pi i)\partial_{\sigma_a}\partial_{\sigma_b}\widetilde{\mathcal{W}}_{\mathrm{eff}}
\end{aligned}
$$

and

$$
-2\pi i\,\partial_{\sigma_a}\partial_{\sigma_b}\widetilde{\mathcal{W}}_{\mathrm{eff}} = \delta_{ab}\left(\frac{1}{\sigma_a - m_i^y - \frac{1}{2}m^z} + \frac{1}{-\sigma_a + m_i^{\widetilde{y}} - \frac{1}{2}m^z} + \sum_{l=1}^{N_c}(\mathcal{S}_{lb}-\mathcal{S}_{bl})\right) - \mathcal{S}_{ab} - \mathcal{S}_{ba}
$$

with

$$
\mathcal{S}_{kl} = \frac{1}{\sigma_k - \sigma_l + m^z}
$$

In $P_{2\mathrm{d}}$, we removed solutions which are same up to the permutations of $(\sigma_1,\cdots,\sigma_{N_c})$.

Gauge-Bethe Correspondence

- Gauge-Bethe Correspondence:

supersymmetric gauge theories

and

quantum integrable system

[Nekrasov-Shatashvili '09]

- It states that

the SUSY parameter condition	$\leftrightarrow$	Bethe Ansatz equation
effective twisted superpotential	$\leftrightarrow$	Yang-Yang function

- Correspondence [Nekrasov-Shatashvili '09]

With

$$\begin{aligned} N_c &= N \\ N_f &= M \\ m^y/m^z = m^{\tilde{y}}/m^z &= \nu/ic \\ (-1)^{N_f} e^{2\pi i\tau} &= e^{i\vartheta} \end{aligned}$$

the SUSY parameter condition	$\leftrightarrow$	Bethe Ansatz equation
effective twisted superpotential	$\leftrightarrow$	Yang-Yang function

reminder:

- $$\prod_{j=1}^{M} \frac{\lambda_a - \nu_j - i\frac{c}{2}}{\lambda_a - \nu_j + i\frac{c}{2}} = e^{i\vartheta} \prod_{\substack{b=1 \\ b\neq a}}^{N} \frac{\lambda_b - \lambda_a + ic}{\lambda_b - \lambda_a - ic} ,$$

- $$\begin{aligned} \widetilde{\mathcal{W}}_{\text{eff}} = \tau \sum_{a=1}^{N_c} \sigma_a - \frac{1}{2} \sum_{1\leq a<b\leq N_c} (\sigma_a - \sigma_b) - \frac{1}{2\pi i} \Bigg[\sum_{a=1}^{N_c} \sum_{i=1}^{N_f} (\sigma_a - m_i^y - \frac{1}{2} m^z)(\log(\sigma_a - m_i^y - \frac{1}{2} m^z) - 1) \\ + (-\sigma_a + m_i^{\tilde{y}} - \frac{1}{2} m^z)(\log(-\sigma_a + m_i^{\tilde{y}} - \frac{1}{2} m^z) - 1) + \sum_{a,b=1}^{N_c} (\sigma_a - \sigma_b + m^z)(\log(\sigma_a - \sigma_b + m^z) - 1) \Bigg] . \end{aligned}$$

Such correspondence with explicit identification was only made at the level of vacua and effective twisted superpotential.

We calculated partition function Z^{2d} of topologically twisted version (A-type) of above 2d $\mathcal{N} = (2,2)$ on S^2
and checked that it equals to the inverse norm of the Bethe eigenstate

$$Z^{2d} = \sum_{(\lambda)\in P_{\mathrm{XXX}}} \langle \Psi_N(\lambda) | \Psi_N(\lambda) \rangle^{-1}$$

where P_{XXX} is the set of the independent solutions of $(\lambda) = (\lambda_1, \ldots, \lambda_N)$ satisfying the Bethe Ansatz equations. *c.f.* [Okuda-Yoshida '12, Nekrasov-Shatashvili '14]

From above correspondence, we also have that

correlation functions of gauge invariant operator $\mathcal{O}(\sigma)$	$\leftrightarrow$	the coefficient of expectation value of Baxter Q-operator $\mathbf{Q}(x)$ whose eigenvalue is $Q(x) = \prod_{a=1}^{N_c}(x - \lambda_a)$

The eigenvalue of transfer matrix $\tau(\mu)$ for $\mathrm{XXX}_{1/2}$ model

$$\theta\left(\mu, \{\lambda_a\}\right) = \mathsf{a}(\mu) \prod_{a=1}^{N} f\left(\mu, \lambda_a\right) + e^{i\vartheta} \mathsf{d}(\mu) \prod_{a=1}^{N} f\left(\lambda_a, \mu\right)$$

which is also expressed in terms of symmetric polynomial of λ_a.

The eigenvalue of transfer matrix $\tau(\mu)$ is generating function of mutually commuting conserved charges (or Hamiltonians).

Accordingly, one can match the expectation value of conserved charges of $\mathrm{XXX}_{1/2}$ spin chain model with the twisted GLSM correlators with appropriate coefficients.

Equivariant Quantum Cohomology

- Non-linear Sigma Model:

  If FI parameter $\xi > 0$,

  the IR theory of 2d $\mathcal{N} = (2,2)^*$ theory (with superpotential $W_{\widetilde{Q}\Phi Q} = \sum_{a,b=1}^{N_c} \sum_{i=1}^{N_f} \widetilde{Q}_i^a \Phi_a{}^b Q_b^i$) is described by non-linear sigma model with hyper-Kähler target space, which is the cotangent bundle of Grassmannian $T^*\mathrm{Gr}(N_c, N_f)$

- Equivariant Quantum Cohomology Algebra
  - Usual cup product of cohomology ring is replaced by “quantum” multiplication between generators of cohomology $H^*(X)$ of target space X
  - It can be defined in terms of Gromov-Witten invariants
  - Equivariant integration of equivariant quantum cohomology class was shown to agree with correlation functions in GLSM for several example.
  - We worked on the cotangent bundle of Grassmannian $T^*\mathrm{Gr}(N_c, N_f)$

Some detail: [Gorbounov-Rimányi-Tarasov-Varchenko '13]

- Equivariant Quantum Cohomology of the Cotangent Bundle of Grassmannian

$$QH^*_{GL_n(\mathbb{C})\times\mathbb{C}^*}(T^*\mathrm{Gr}(r,n);\mathbb{C}) \,=\, \mathbb{C}[\mathbf{z},\Gamma,h]^{S_n\times S_{\lambda_1}\times S_{\lambda_2}} \otimes \mathbb{C}[[\mathsf{q}]]\,/\,\mathcal{I}_{\mathsf{q}}$$

  - $G(r,n)$: $\quad 0 \,=\, F_0 \,\subset\, F_1 \,\subset\, F_2 \,=\, \mathbb{C}^n$ with $\dim F_1 = r$.
  - $(\lambda_1,\lambda_2) := (r, n-r)$
  - $\Gamma_i = \{\gamma_{i,1},\cdots,\gamma_{i,\lambda_i}\}$ with $i=1,2$: the set of Chern roots of bundles on $\mathrm{Gr}(r,n)$ with fiber F_i/F_{i-1}
  - $\mathbf{z}=\{z_1;\cdots;z_n\}$: Chern roots corresponding to each factors of $(\mathbb{C}^*)^n \subset GL(n,\mathbb{C})$ action
  - h: the Chern root corresponding to $\mathbb{C}^*$ action on the fiber direction of $T^*\mathrm{Gr}(r,n)$
  - S_n, S_{λ_1} and S_{λ_2}: the symmetrization of variables $\{z_1,\cdots,z_n\}$, $\{\gamma_{1,1},\cdots,\gamma_{1,\lambda_1}\}$ and $\{\gamma_{2,1},\cdots,\gamma_{2,\lambda_2}\}$, respectively.
  - $\mathbb{C}[[\mathsf{q}]]$ is the ring of formal series of q, which is the quantum parameter

- ideal $\mathcal{I}_{\mathrm{q}}$ is generated by the n coefficients p_l defined by

$$\sum_{l=1}^{n} p_l(\mathbf{z}, \Gamma, h, \mathrm{q})u^{n-l} := \prod_{a=1}^{2}\prod_{b=1}^{\lambda_a}(u-\gamma_{a,b}) - \mathrm{q}\prod_{a=1}^{\lambda_1}(u-\gamma_{1,a}-h)\prod_{b=1}^{\lambda_2}(u-\gamma_{2,b}+h) - (1-\mathrm{q})\prod_{i=1}^{n}(u-z_i)$$

The coefficients p_l are degree l polynomials of each Γ and $\mathbf{z}$ and invariant under the action of $S_n \times S_{\lambda_1} \times S_{\lambda_2}$.

Example: $T^*\mathbb{CP}^1$ ($N_c = 1, N_f = 2$)

$$\gamma_{1,1}^2 = (z_1+z_2)\gamma_{1,1} + \frac{2h\mathrm{q}}{1-\mathrm{q}}\gamma_{1,1} + \frac{h\mathrm{q}(h-z_1-z_2)+\mathrm{q}z_1z_2}{1-\mathrm{q}}\,.$$

- Multiplication above are meant to be "quantum" multiplication; $\gamma_{1,1}^2 = \gamma_{1,1} \star \gamma_{1,1}$
- "Classical" or usual cup product of cohomology classes is obtained by $q \to 0$ (large volume limit)

- By massaging the equivariant quantum cohomology relations, we can obtain the SUSY vacua condition

$$\mathrm{q} \prod_{\substack{a=1 \\ a \neq c}}^{\lambda_1} \frac{\gamma_{1,c} - \gamma_{1,a} + h}{\gamma_{1,c} - \gamma_{1,a} - h} = \prod_{i=1}^{n} \frac{\gamma_{1,c} - z_i}{\gamma_{1,c} - z_i + h}$$

- From this, we have an identification of parameters between GLSM and quantum cohomology

$$r = N_c, \quad n = N_f, \quad \gamma_{1,a} = \sigma_a, \quad z_i = m_i + \frac{m^z}{2}, \quad h = m^z, \quad \mathrm{q} = (-1)^{N_f} q = (-1)^{N_f} e^{2\pi i \tau}.$$

- "Classical" equivariant integration formula for $\langle\gamma\rangle := \int_{T^*\mathbb{CP}^1}\gamma$ is available in [Gorbounov-Rimányi-Tarasov-Varchenko '13]
- By using such formula, we calculated the equivariant integration of higher-order power of γ with nonzero q
- We showed that they agree with GLSM correlator for several examples, *e.g.*

$$\langle\sigma^l\rangle^{N_c=1,N_f=2}_{\text{A-twist}} = \langle\gamma^l_{1,1}\rangle_{T^*\mathbb{CP}^1}$$

- It was possible to check Seiberg-like duality between $U(N_c)$ with N_f hypers and $U(N_f - N_c)$ with N_f hypers

$$T^*\mathrm{Gr}(r,n) \cong T^*\mathrm{Gr}(n-r,n)$$

c.f. [Benini-Park-Zhao '14]

- Remark: Quantum cohomology corresponds to the integrable system.

  In [Maulik-Okounkov '12], they construct Yangian acting on the equivariant cohomology of Nakajima quiver varieties.

  In [Gorbounov-Rimányi-Tarasov-Varchenko '13], it was shown that the equivariant quantum cohomology of the cotangent bundle of partial flag varieties is isomorphic to the Bethe subalgebra of $\mathfrak{gl}_N$ XXX spin chain model.

Summary

- We have checked that the norm of Bethe eigenstate agree with partition function of 2d $\mathcal{N} = (2,2)$ theory.
- We also provided a way to calculate equivariant integration of quantum cohomology for $T^*\mathrm{Gr}(r,n)$ and found agreement with GLSM correlator of 2d $\mathcal{N} = (2,2)^*$.
- We also worked on $\mathrm{XXZ}_{1/2}$, topological twisted 3d $\mathcal{N} = 2$ theories on $S^1 \times S^2$, and the equivariant quantum K-theory algebra.